COMPUTATION OF EXPLICIT PREIMAGES IN ONE-DIMENSIONAL CELLULAR AUTOMATA APPLYING THE DEBRUIJN DIAGRAM

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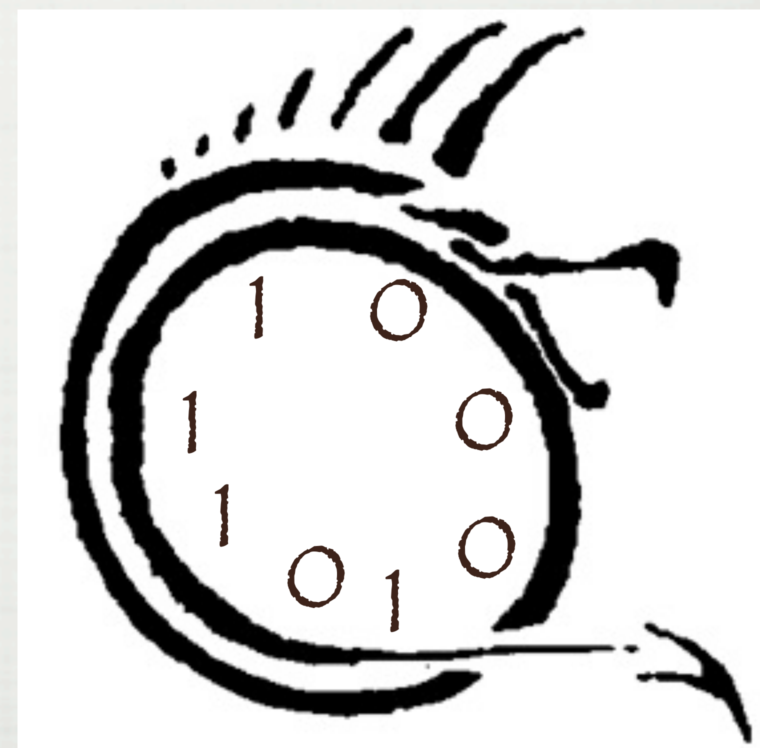
ORIGIN OF THE DEBRUIJN DIAGRAM

N. DEBRIJIN SOLVED THE PROBLEM OF FINDING A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH K.

EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

- 0111010001



EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

- 0111010001

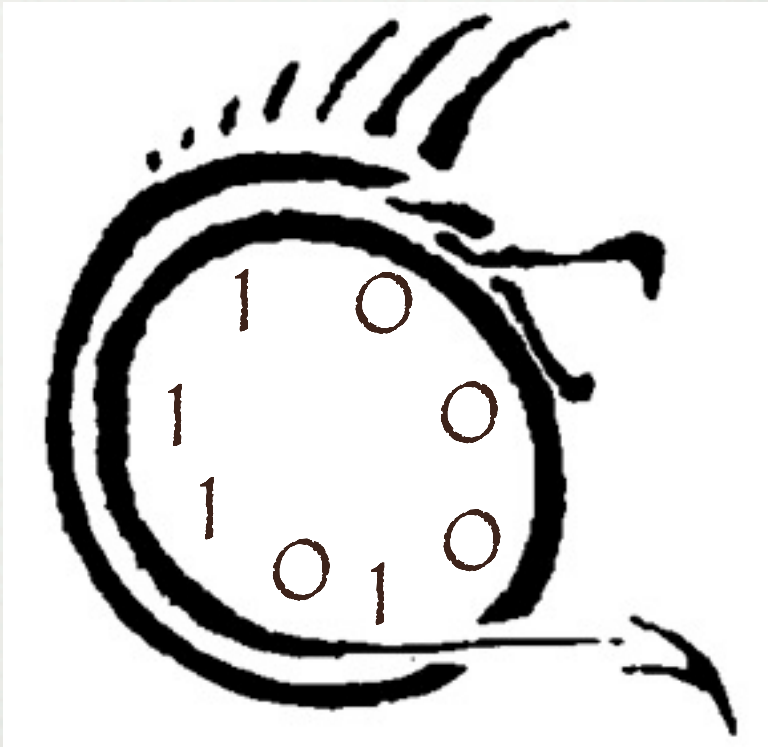
  011

EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

- 0111010001

011
111

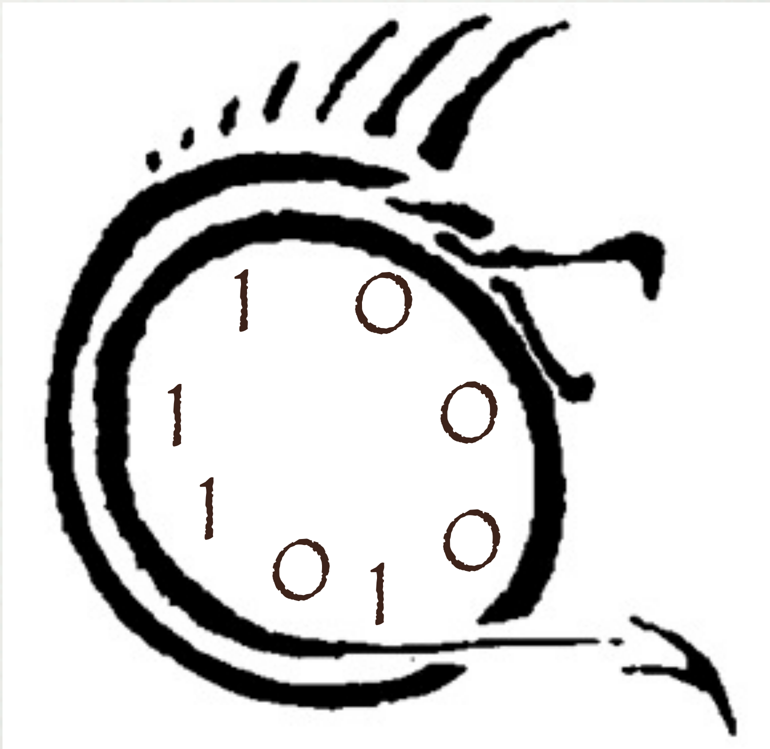


EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

- 0111010001

011
111
110

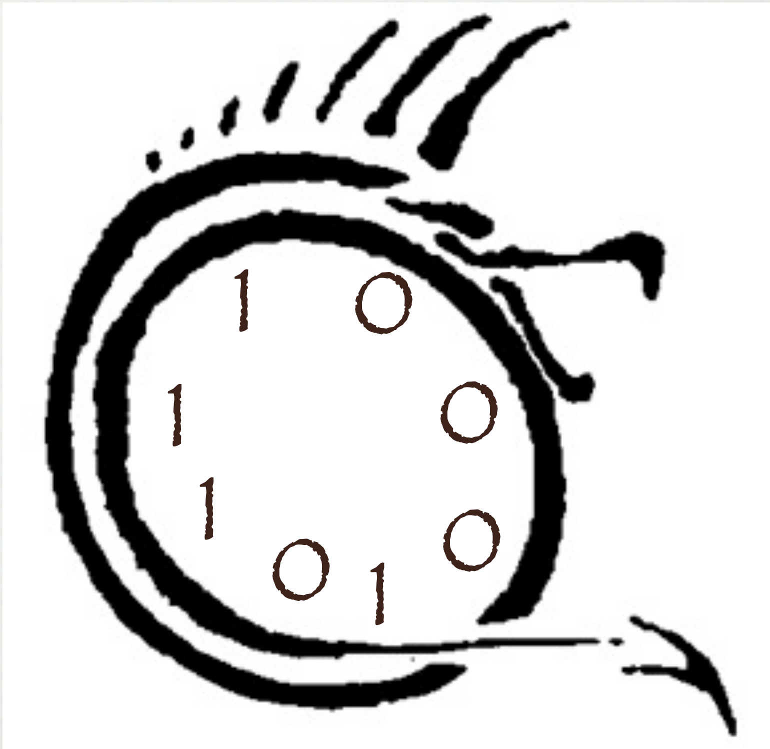


EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

- 0111010001

011
111
110
101



EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

- 0111010001

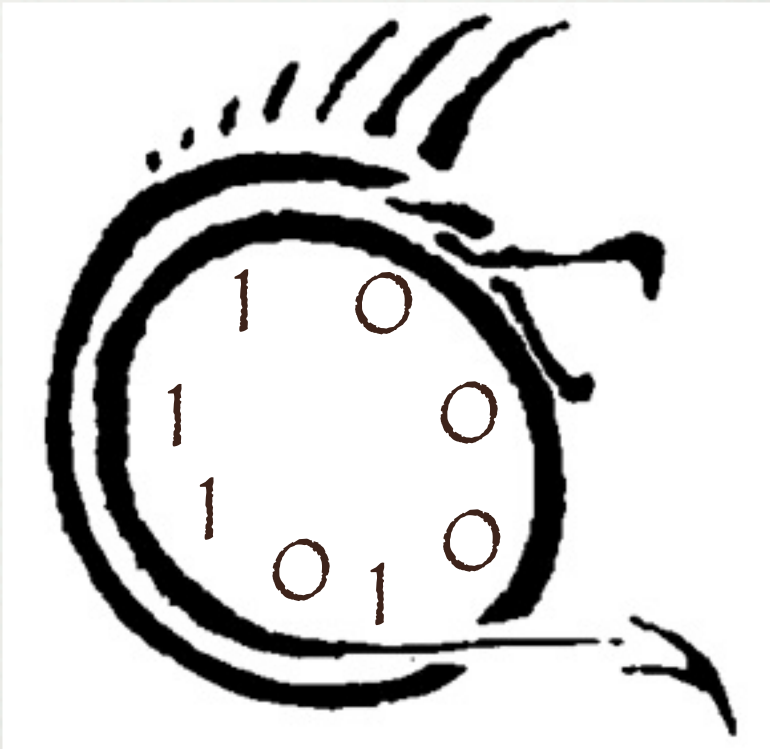
011
111
110
101
010

EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

- 0111010001

011
111
110
101
010
100



EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

- 0111010001

011
111
110
101
010
100
000

EXAMPLE

A MINIMUM-LENGTH BINARY STRING THAT CONTAINS AS A SUBSTRING EVERY BINARY STRING OF A PRESCRIBED LENGTH 3.

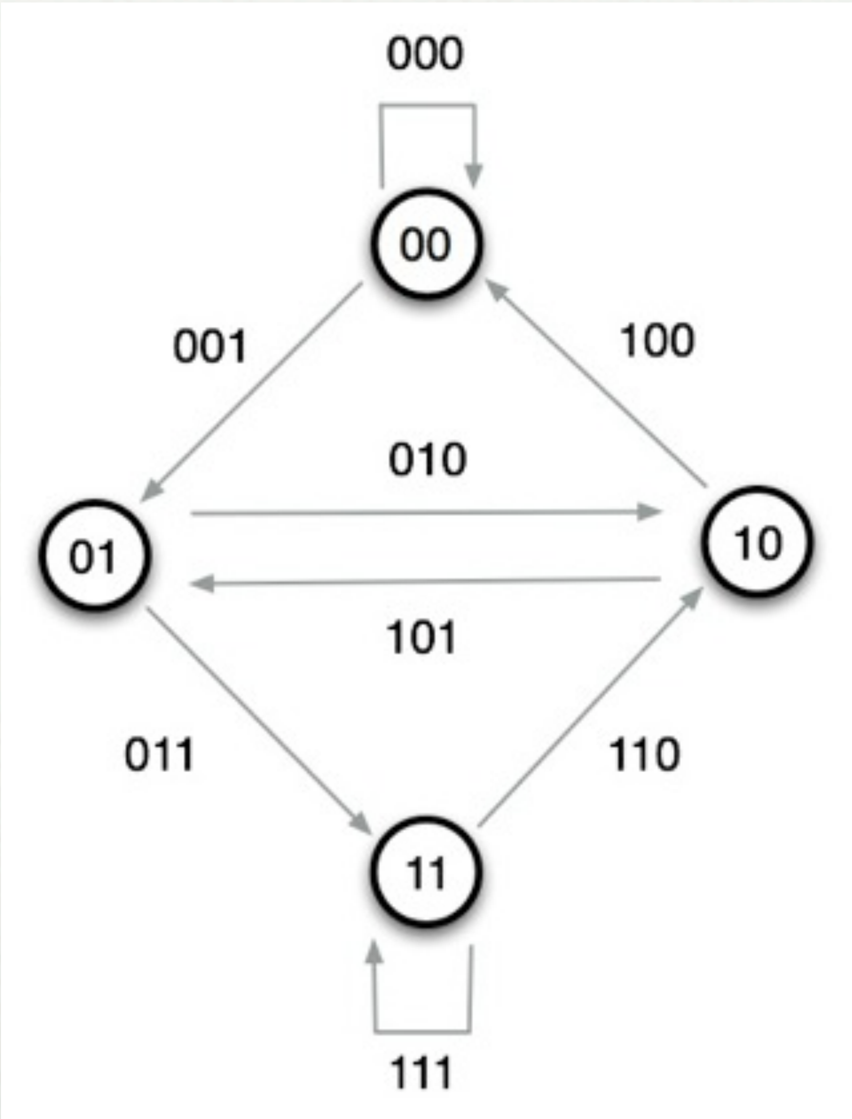
- 0111010001

  011
  111
  110
  101
  010
  100
  000
  001

THE DEBRUIJN DIAGRAM

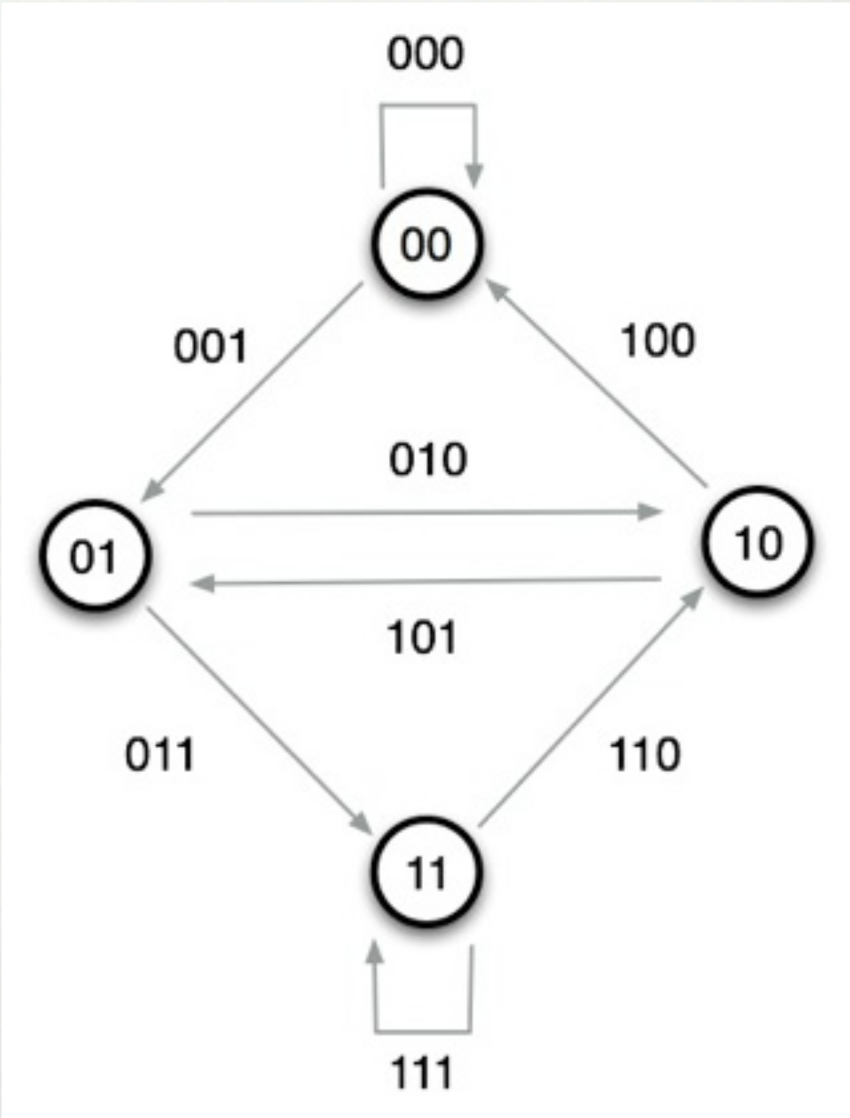
Definition: A deBruijn graph of order k, denoted by $G(k)$, is a directed graph with 2^k vertices, each labeled with a unique k-bit string. Vertex a is joined to vertex b if bitstring b is obtainable from bitstring a by either a cycle shift or a deBruijn shift.

EXAMPLE



THE DEBRUIJN GRAPH

EULERIAN PATH TROUGH THE DEBRUIJIN GRAPH SOLVED THE PROBLEM



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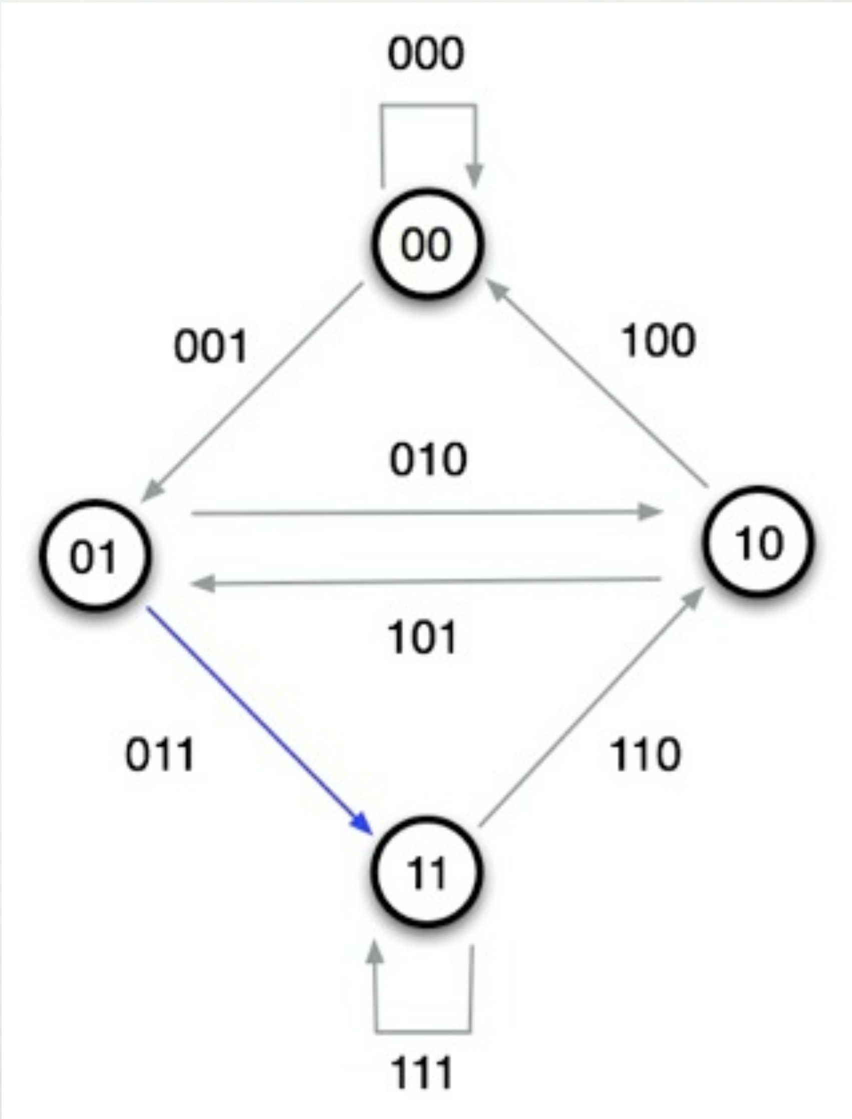
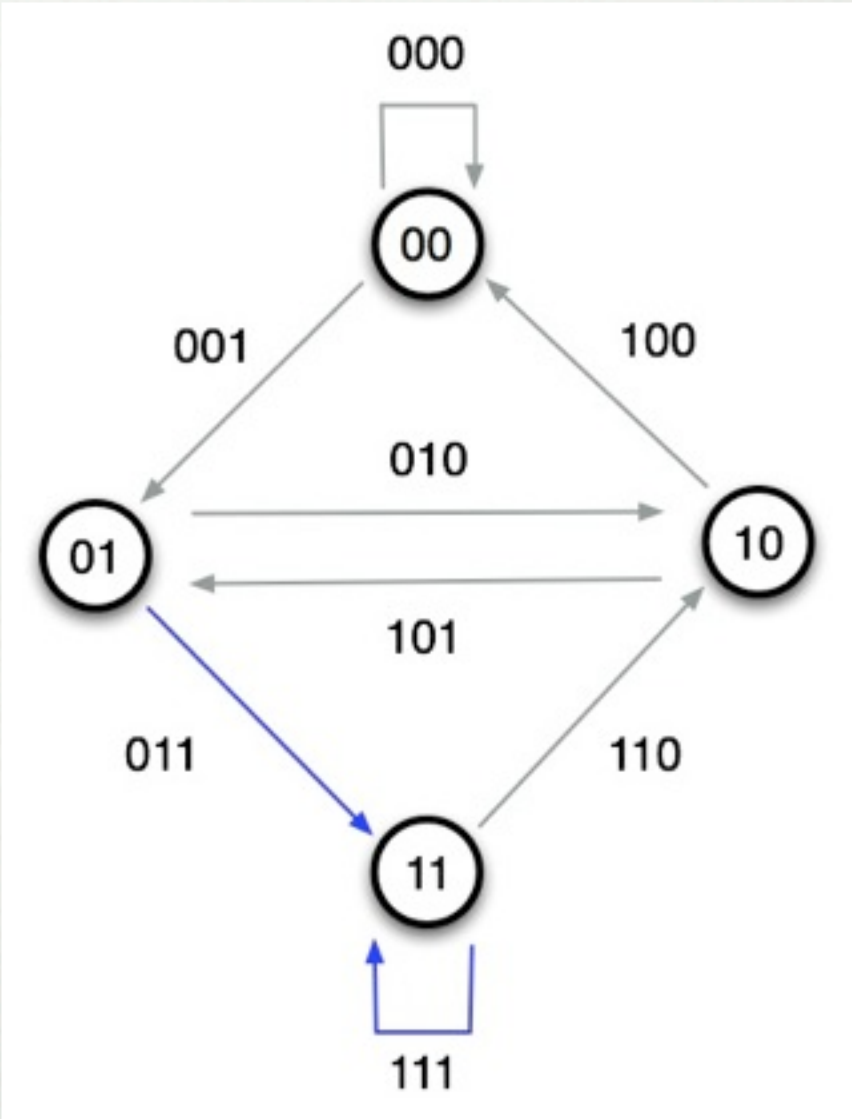
EULERIAN PATH TROUGH THE DEBRUIJIN GRAPH SOLVED THE PROBLEM



011

THE DEBRUIJIN GRAPH

EULERIAN PATH TROUGH THE DEBRUIJIN GRAPH SOLVED THE PROBLEM



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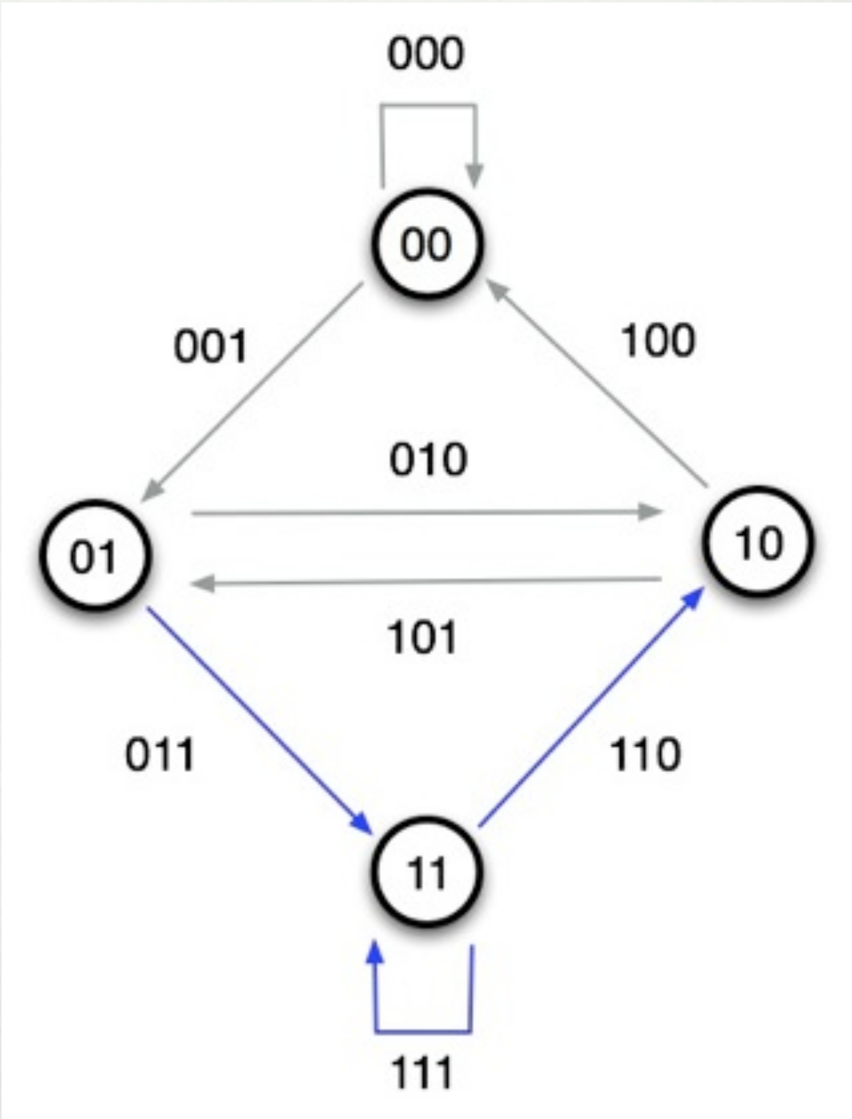
THE DEBRUIJIN GRAPH

EULERIAN PATH TROUGH THE DEBRUIJIN GRAPH SOLVED THE PROBLEM



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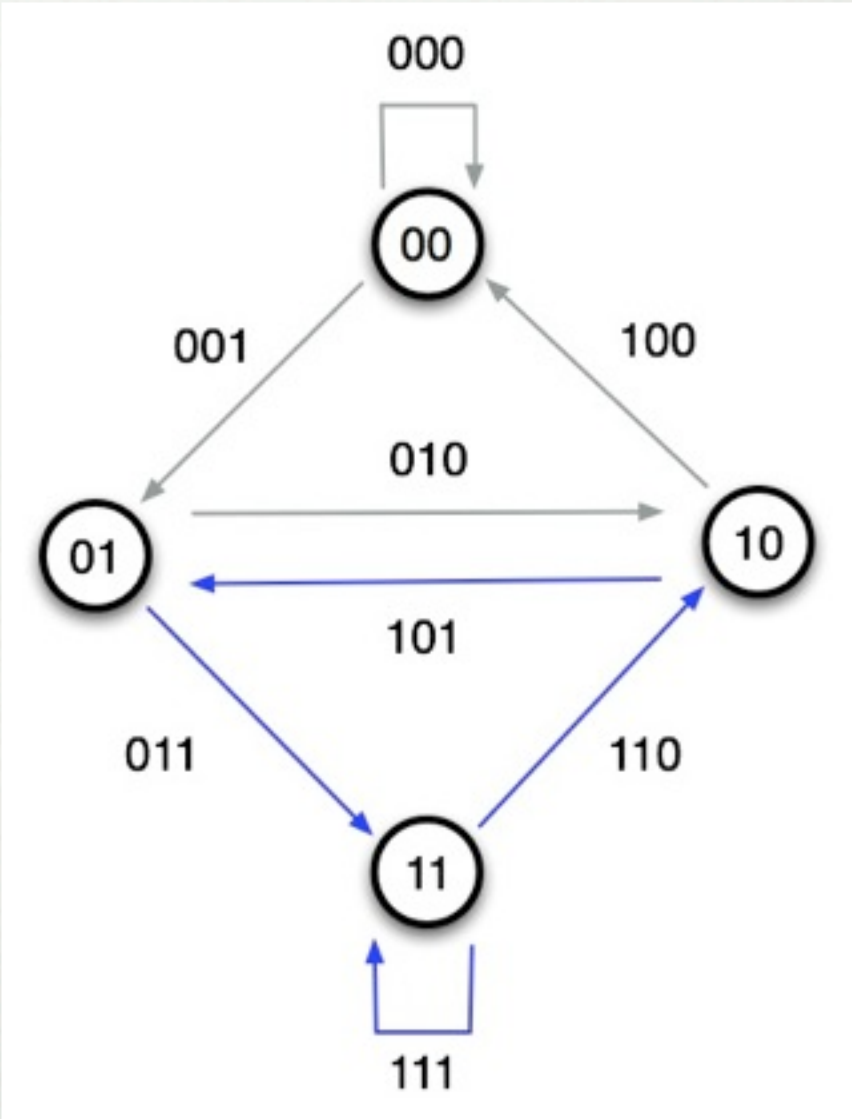
THE DEBRUIJIN GRAPH

EULERIAN PATH TROUGH THE DEBRUIJIN GRAPH SOLVED THE PROBLEM



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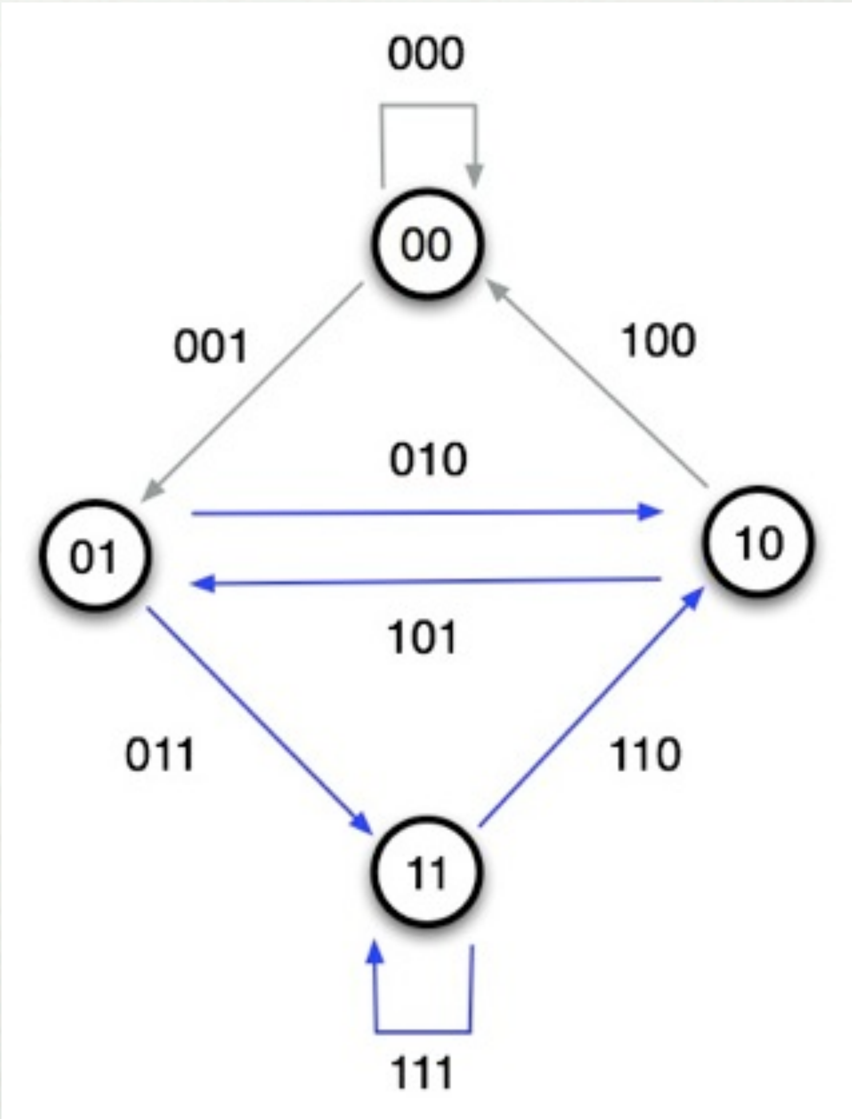
THE DEBRUIJN GRAPH

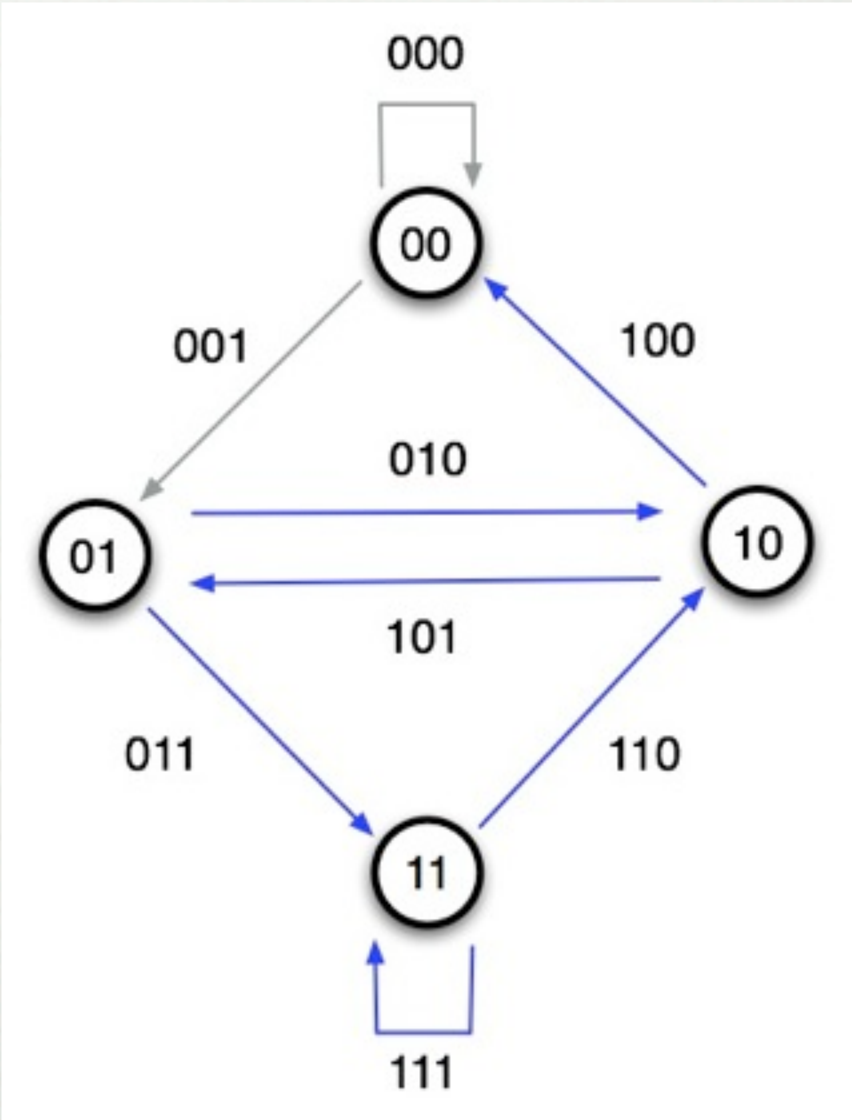
EULERIAN PATH TROUGH THE DEBRUIJN GRAPH SOLVED THE PROBLEM



0111010

THE DEBRUIJIN GRAPH

EULERIAN PATH TROUGH THE DEBRUIJIN GRAPH SOLVED THE PROBLEM



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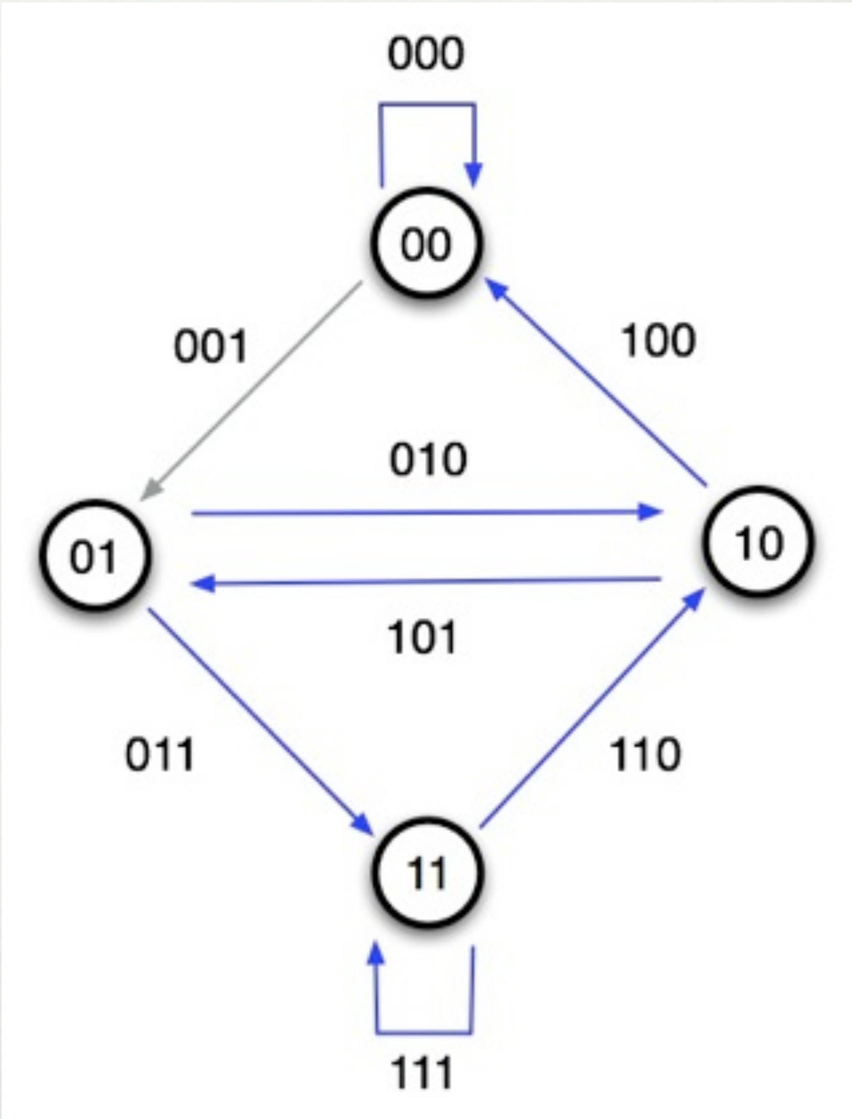
THE DEBRUIJIN GRAPH

EULERIAN PATH TROUGH THE DEBRUIJIN GRAPH SOLVED THE PROBLEM



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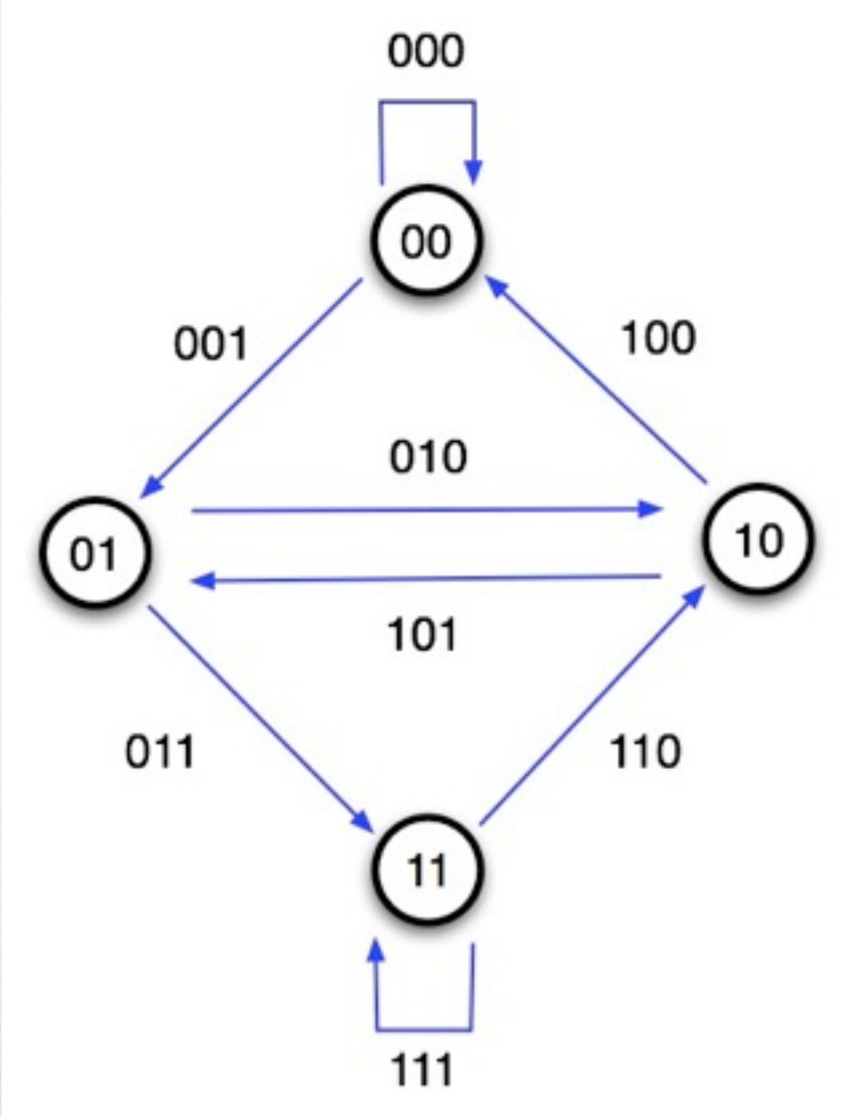
THE DEBRUIJIN GRAPH

EULERIAN PATH TROUGH THE DEBRUIJIN GRAPH SOLVED THE PROBLEM



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CELLULAR AUTOMATA

A one-dimensional cellular automaton is a discrete dynamical system it consist in a quintuple, $\{\Sigma, \Phi, \varphi, \eta_r(x_i), c_0\}$, wherein:

- Σ is a finite set of states, from which the configurations of c cells take their values, $c : \mathbb{Z} \rightarrow \Sigma$.
- $\eta_r(x_i) = x_{i-r}, \ldots, x_i, \ldots, x_{i+r}$ is the neighborhood of x_i of radius r, whose size is $\tau = |\eta_r(x_i)|$.
- $\varphi : \Sigma^\tau \rightarrow \Sigma$, a local function which maps neighborhoods with size τ to a set of states Σ.
- C_0, an initial configuration which is the starting point of the evolution.
- Φ is a global function that computes transformations between sets of configurations.

EXAMPLE

$$\Sigma = \{ \square, \blacksquare \} \qquad r = 1$$

EXAMPLE

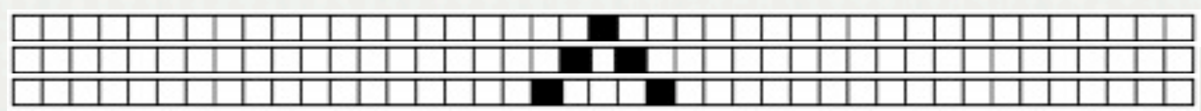
$$\sum = \{ \square , \blacksquare \}$$

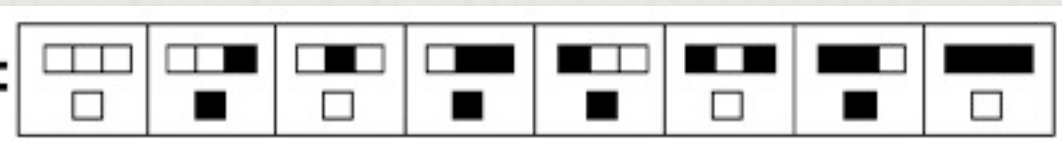
$$r = 1$$

EXAMPLE

$$\sum = \{\square, \blacksquare\}$$

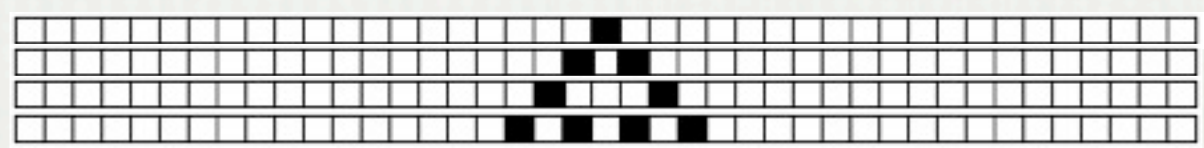
$$r = 1$$



$\varphi =$ 

EXAMPLE

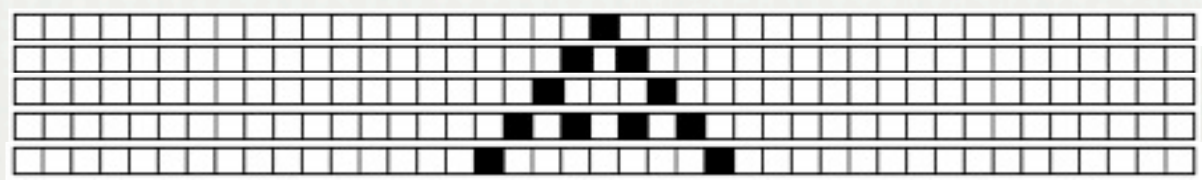
$\sum = \{\square, \blacksquare\}$ $\quad\quad r = 1$



EXAMPLE

$$\sum = \{ \square, \blacksquare \}$$

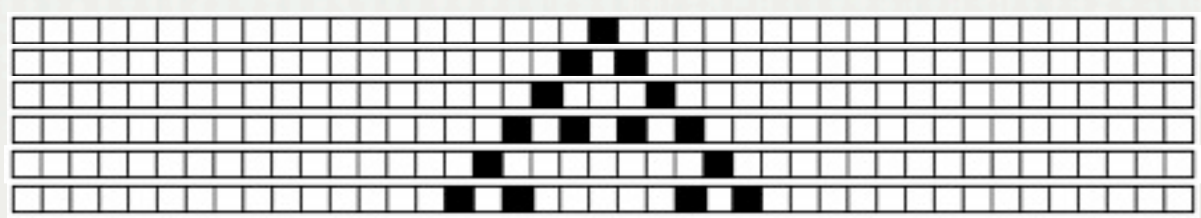
$$r = 1$$



$\varphi =$ 

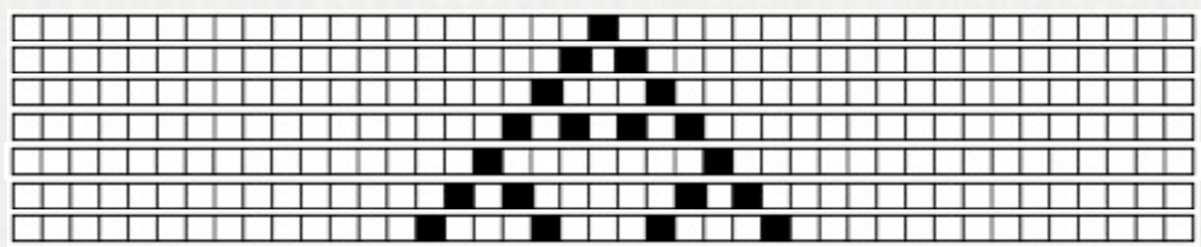
EXAMPLE

$$\sum = \{ \square, \blacksquare \} \qquad r = 1$$

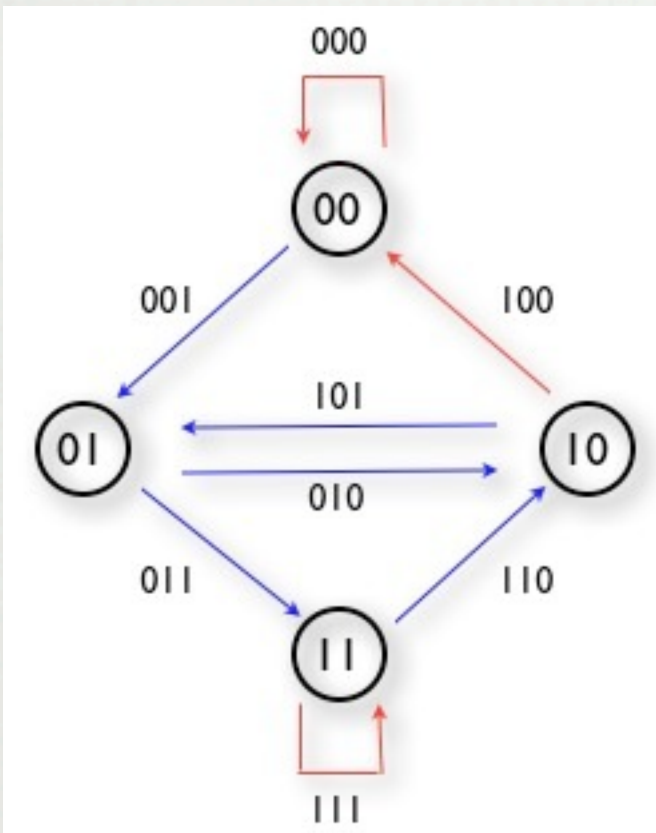


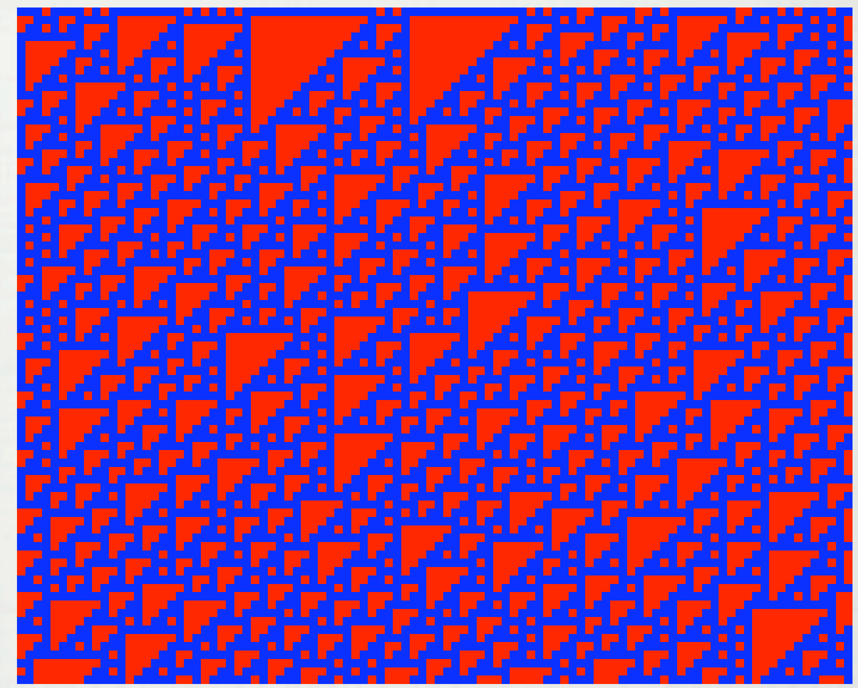
EXAMPLE

$$\Sigma = \{\square, \blacksquare\} \qquad r = 1$$

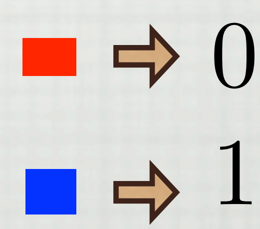


THE DEBRUIJIN DIAGRAM AND CELLULAR AUTOMATA





$$\varphi = \{\ 000,100,111 \to 0,\ 001,010,011,101,110 \to 1\ \}$$



PREIMAGES

The preimages of a single cell are the locally valid neighborhoods defined by the inverse of the local transition function

$$\varphi^{-}1(c_x^t) = \{n_x^t - 1 \in \Sigma^N | \varphi(n_x^{t-1} = c_v^t)$$

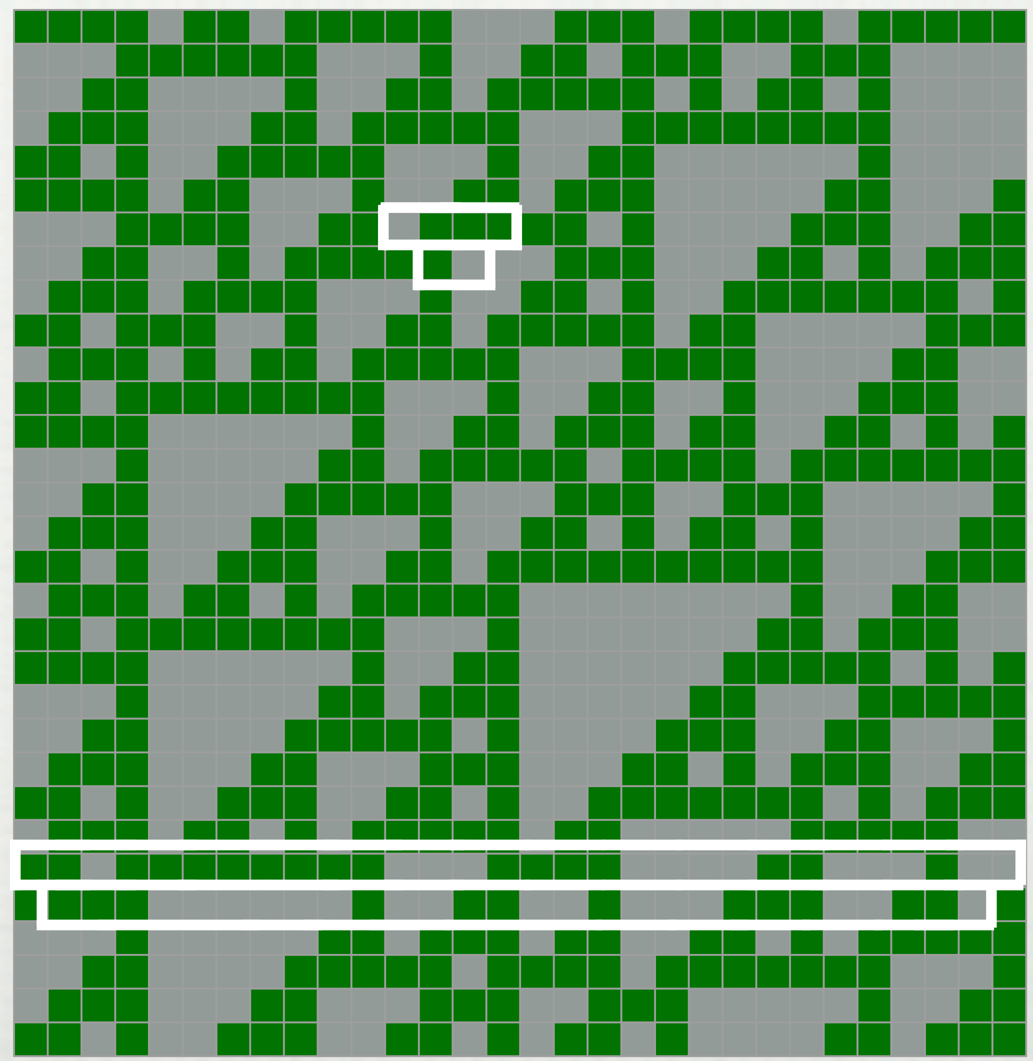
Preimages C^{t-1} of a block m are defined by the inverse of the local transition function

$$\varphi^{-}1(c_x^t) = \{m_x^t - 1 \in \Sigma^N | \varphi(m_x^{t-1} = c_v^t)$$

Locally valid neighborhoods of adjacent cells must overlap correctly to become m block valid.

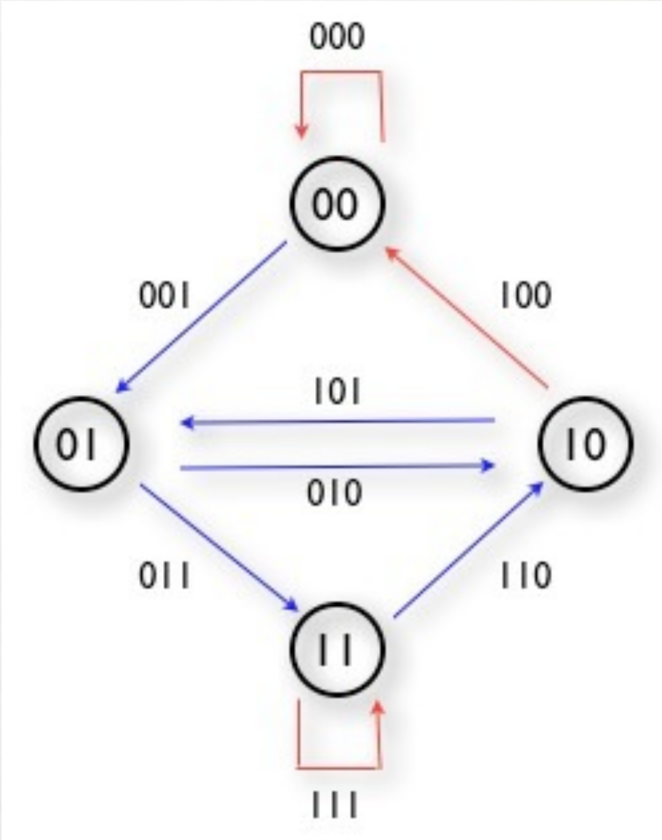
EXAMPLE OF PREIMAGE

- ■ ⇨ 0
- ■ ⇨ 1



PREIMAGES AND THE BRUIJIN GRAPH

LABELING EDGES
AS
NEIGHBORHOODS
AND AS
NEIGHBORHOODS
MAPPING



0111
10

■ ⇨ 0

■ ⇨ 1

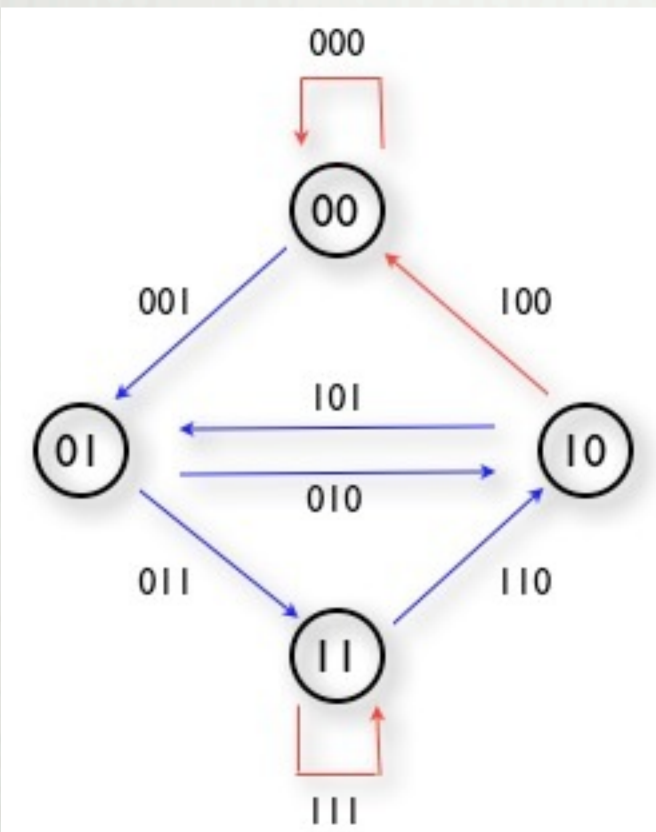
MATRIX REPRESENTATION OF THE DEBRUIJN GRAPH

Let v_i and $v_j \in V(G)$ of the *De Bruijn diagram* for $i, j = 1, 2, 3, \ldots, |V(G)|$. The preimages matrix $M(s)_{i,j}$ of state $s \in \Sigma$ is defined as:

$$M(s)_{i,j} = \begin{cases} \{N(v_i v_j)\} & \text{If } \phi(N(v_i, v_j)) = s \text{ where } s \in \Sigma \\ \emptyset & \text{elsewhere} \end{cases} \tag{1}$$

where its element sets are neighborhood that represent v_i and v_j for $i, j = 1 \ldots |V(G)|$ it means $N(v_i, v_j)$, where the mapping corresponds to state $s \in \Sigma$. To simplify the notation $M(s)_{i,j}$ is denoted as M_s.

PREIMAGES MATRIX STATE 0

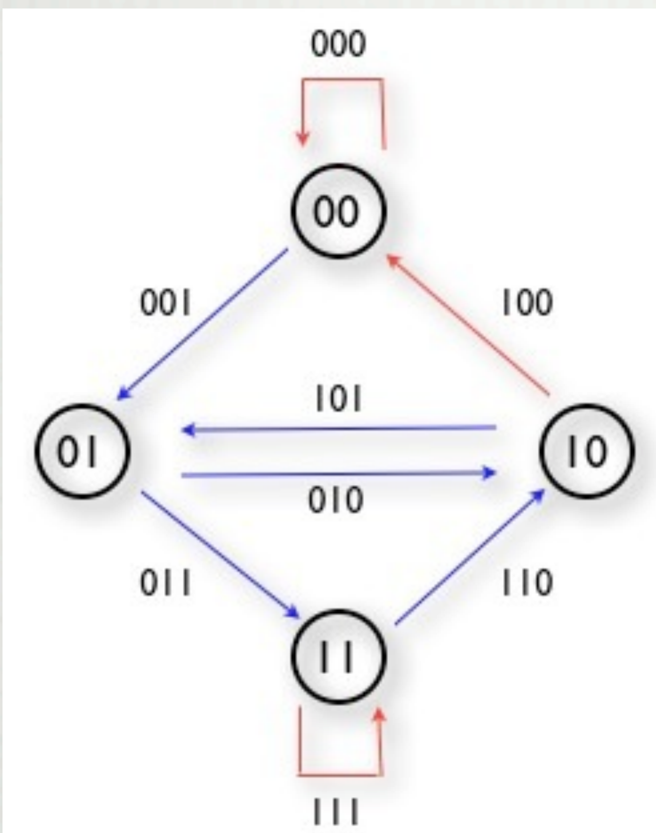


$$M_0 = \begin{pmatrix} \{\text{“000”}\} & \emptyset & - & - \\ - & - & \emptyset & \emptyset \\ \{\text{“100”}\} & \emptyset & - & - \\ - & - & \emptyset & \{\text{“111”}\} \end{pmatrix}$$

■ ⇨ 0

■ ⇨ 1

PREIMAGES MATRIX STATE 0

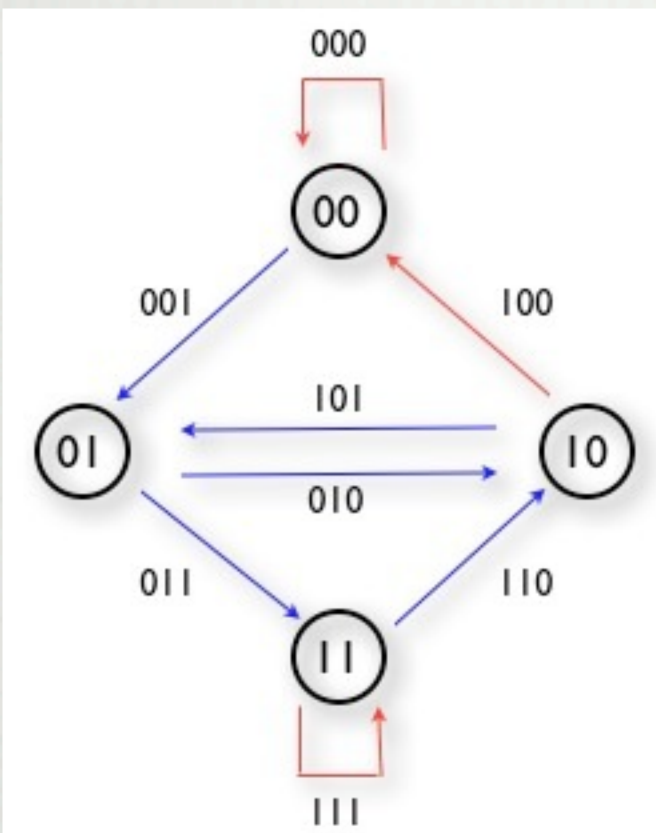


$$
\begin{array}{cc}
 & \begin{array}{cccc} 00 & & & \end{array} \\
M_0 = & \begin{pmatrix} \{\text{“000”}\} & \emptyset & - & - \\ - & - & \emptyset & \emptyset \\ \{\text{“100”}\} & \emptyset & - & - \\ - & - & \emptyset & \{\text{“111”}\} \end{pmatrix}
\end{array}
$$

⇨ 0

⇨ 1

PREIMAGES MATRIX STATE 0

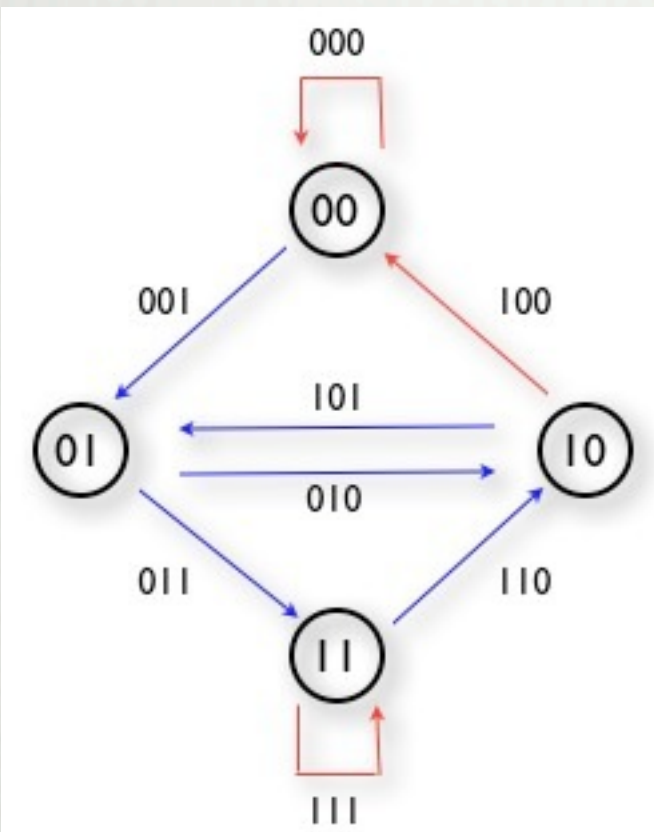


$$
\begin{array}{cc}
 & \begin{array}{cccc} 00 & 01 & & \end{array} \\
M_0 = & \begin{pmatrix}
\{\text{“000”}\} & \emptyset & - & - \\
- & - & \emptyset & \emptyset \\
\{\text{“100”}\} & \emptyset & - & - \\
- & - & \emptyset & \{\text{“111”}\}
\end{pmatrix}
\end{array}
$$

⇨ 0

⇨ 1

PREIMAGES MATRIX STATE 0

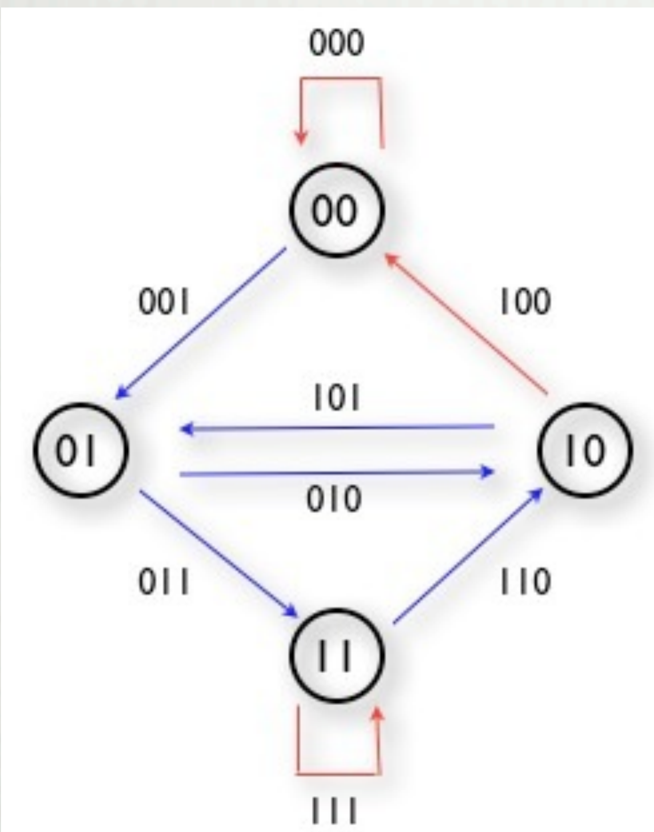


$$
\begin{array}{cc}
 & \begin{array}{cccc} 00 & 01 & 10 & \phantom{\{\text{“111”}\}} \end{array} \\
M_0 = & \begin{pmatrix}
\{\text{“000”}\} & \emptyset & - & - \\
- & - & \emptyset & \emptyset \\
\{\text{“100”}\} & \emptyset & - & - \\
- & - & \emptyset & \{\text{“111”}\}
\end{pmatrix}
\end{array}
$$

⇨ 0

⇨ 1

PREIMAGES MATRIX STATE 0

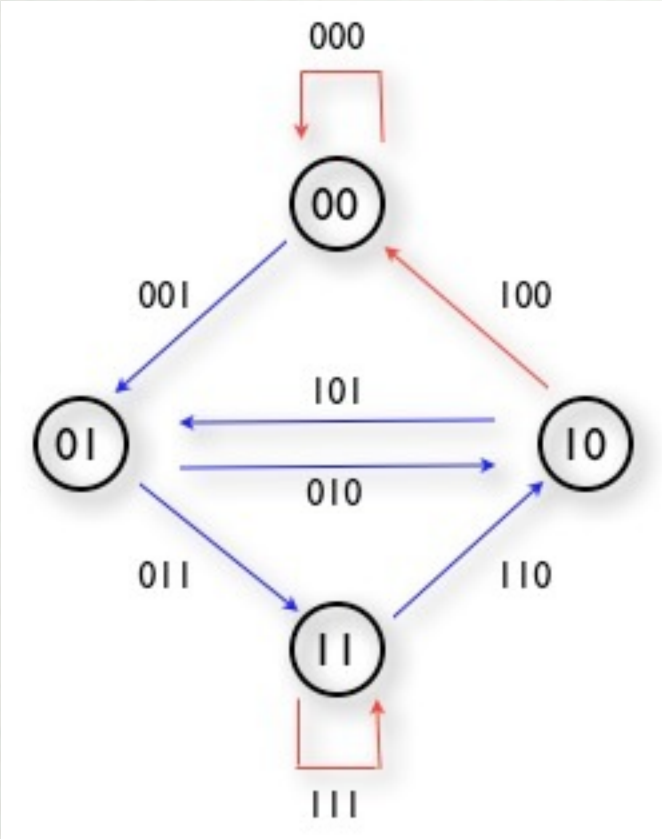


$$
\begin{array}{c} \\ M_0 = \end{array}
\begin{array}{c}
\begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\
\begin{pmatrix}
\{\text{“000”}\} & \emptyset & - & - \\
- & - & \emptyset & \emptyset \\
\{\text{“100”}\} & \emptyset & - & - \\
- & - & \emptyset & \{\text{“111”}\}
\end{pmatrix}
\end{array}
$$

■ ⇨ 0

■ ⇨ 1

PREIMAGES MATRIX STATE 0

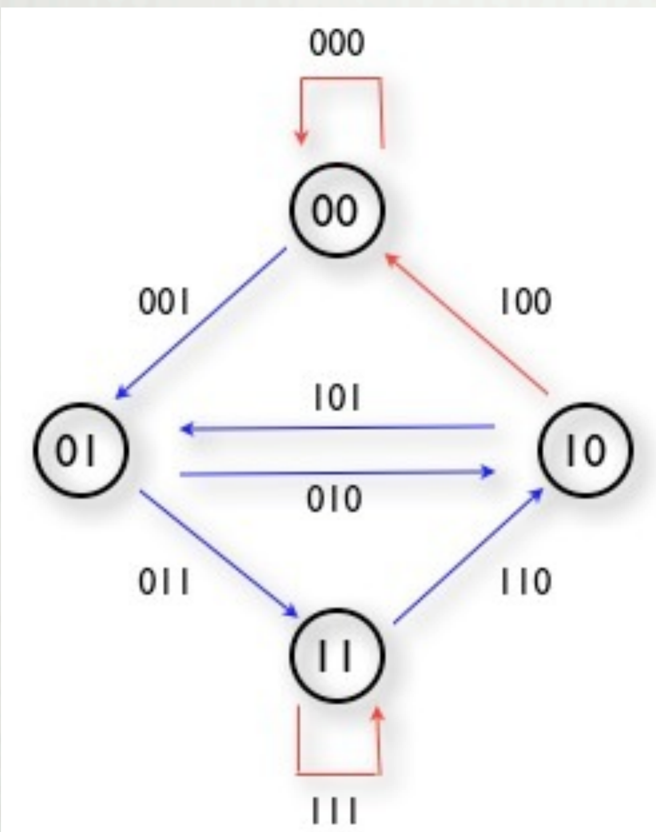


$$M_0 = \begin{array}{cc} & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ 00 & \begin{pmatrix} \{\text{“000”}\} & \emptyset & - & - \\ - & - & \emptyset & \emptyset \\ \{\text{“100”}\} & \emptyset & - & - \\ - & - & \emptyset & \{\text{“111”}\} \end{pmatrix} \end{array}$$

■ ⇨ 0

■ ⇨ 1

PREIMAGES MATRIX STATE 0

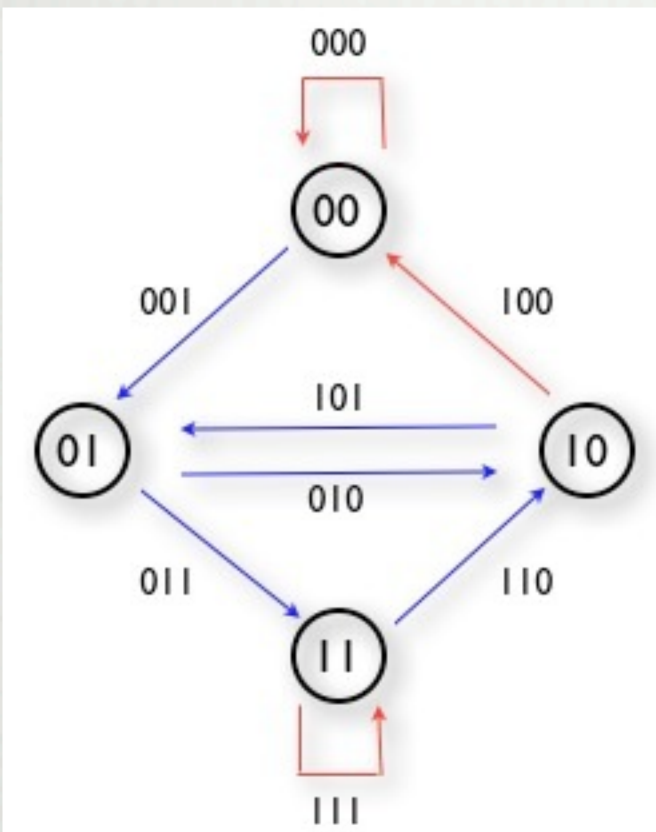


$$M_0 = \begin{array}{cc} & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{array}{c} 00 \\ 01 \\ \\ \\ \end{array} & \begin{pmatrix} \{\text{“000”}\} & \emptyset & - & - \\ - & - & \emptyset & \emptyset \\ \{\text{“100”}\} & \emptyset & - & - \\ - & - & \emptyset & \{\text{“111”}\} \end{pmatrix} \end{array}$$

⇨ 0

⇨ 1

PREIMAGES MATRIX STATE 0

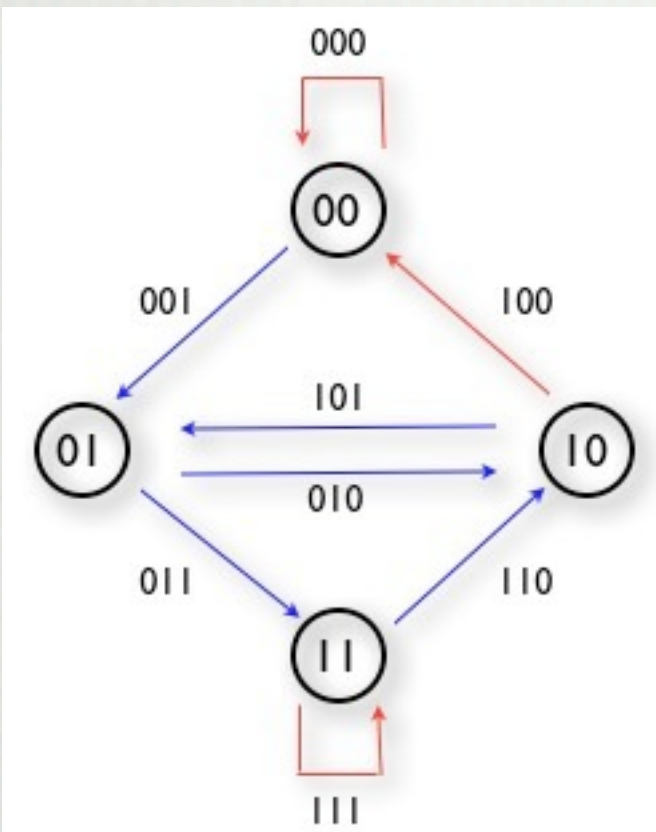


$$
\begin{array}{cc}
 & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\
M_0 = \begin{array}{c} 00 \\ 01 \\ 10 \\ \end{array} & \begin{pmatrix} \{\text{“000”}\} & \emptyset & - & - \\ - & - & \emptyset & \emptyset \\ \{\text{“100”}\} & \emptyset & - & - \\ - & - & \emptyset & \{\text{“111”}\} \end{pmatrix}
\end{array}
$$

$\Rightarrow 0$

$\Rightarrow 1$

PREIMAGES MATRIX STATE 0

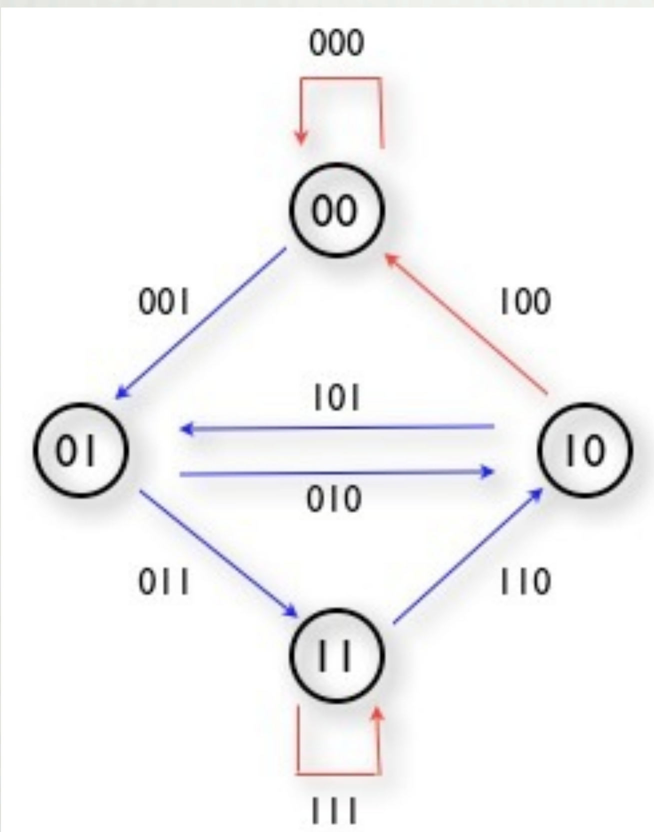


$$M_0 = \begin{array}{c} \\ 00 \\ 01 \\ 10 \\ 11 \end{array} \begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} \{\text{“000”}\} & \emptyset & - & - \\ - & - & \emptyset & \emptyset \\ \{\text{“100”}\} & \emptyset & - & - \\ - & - & \emptyset & \{\text{“111”}\} \end{pmatrix} \end{array}$$

⇨ 0

⇨ 1

PREIMAGES MATRIX STATE 1



$$
M_1 = \begin{array}{c} \\ 00 \\ 01 \\ 10 \\ 11 \end{array}
\begin{array}{c}
\begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\
\begin{pmatrix}
\emptyset & \{\text{“001”}\} & - & - \\
- & - & \{\text{“010”}\} & \{\text{“011”}\} \\
\emptyset & \{\text{“101”}\} & - & - \\
- & - & \{\text{“110”}\} & \emptyset
\end{pmatrix}
\end{array}
$$

$\Rightarrow 0$

$\Rightarrow 1$

OPERATOR BETWEEN MATRICES TO CALCULATE PREIMAGES