Computación Universal en Autómatas Celulares en 2D

Encuentro Internacional de Matemáticas.
Barranquilla Colombia, Octubre 2017.

José Manuel Gómez Soto
Andrew Wuensche

November 24, 2017

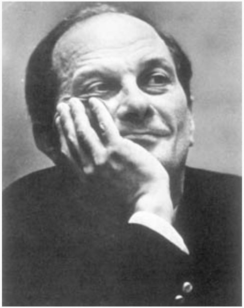
Unidad Académica de Matemáticas
Universidad Autónoma de Zacatecas
Zacatecas, Zac. México.

Contenido

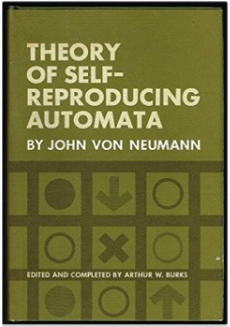
Máquinas que se autoreproducen.

John von Neumann

Stalisnaw Ulam



Theory of self-reproducing automata



Autómata Celular de Von Neumann

- 29 estados, Vecindad de Moore (John von Neumann 1966)
- 8 estados, Vecindad Von Neumann (Edgar Cood 1968)
- 4 estados, Vecindad Von Neumann (Edwin Roger Banks 1970)
- 2 estados, Vecindad Moore (John Horton Conway 1970)

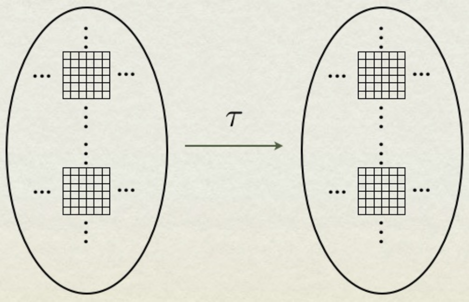
Video

Video

Autómata Celular

$$G = \mathbb{Z}^2,\ A = \{0, 1\}$$

$$\tau = A^{\mathbb{Z}^2} \to A^{\mathbb{Z}^2}$$



S: Vecindad de Moore

$$S = \{x_{i-1,j+1}, x_{i,j+1}, x_{i+1,j+1}, x_{i-1,j}, x_{i,j}, x_{i+1,j}, x_{i-1,j-1}, x_{i,j-1}, x_{i+1,j-1}\}$$

$x_{i-1,j+1}$	$x_{i,j+1}$	$x_{i+1,j+1}$
$x_{i-1,j}$	$x_{i,j}$	$x_{i+1,j}$
$x_{i-1,j-1}$	$x_{i,j-1}$	$x_{i+1,j-1}$

S: Vecindad de von Neumann

$$S = \{x_{i,j+1}, x_{i-1,j}, x_{i,j}, x_{i+1,j}, x_{i,j-1}\}$$

	$x_{i,j+1}$	
$x_{i-1,j}$	$x_{i,j}$	$x_{i+1,j}$
	$x_{i,j-1}$	

μ

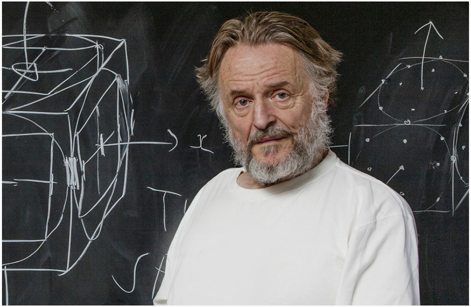
$$\mu = A^S \to A$$

$$x_{i,j}^t t + 1 = \mu(x_{i-1,j+1}^t, x_{i,j+1}^t, x_{i+1,j+1}^t, x_{i-1,j}^t, x_{i,j}^t, x_{i+1,j}^t, x_{i-1,j-1}^t, x_{i,j-1}^t, x_{i+1,j-1}^t)$$

El Juego de la Vida

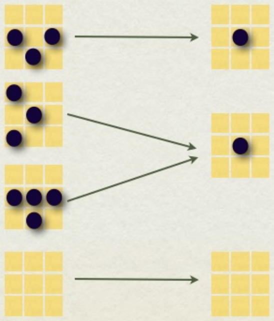
Juego de la Vida

$$G = \mathbb{Z}^2,\ A = \{0, 1\}$$



Juego de la Vida

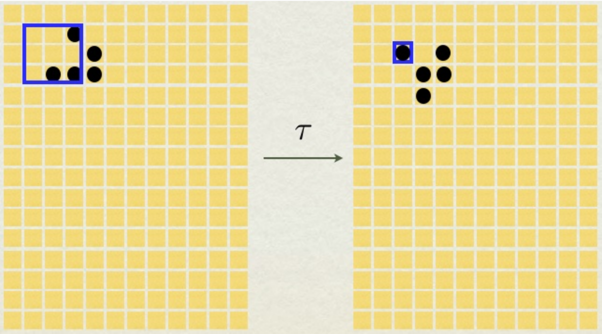
$$G = \mathbb{Z}^2,\ A = \{0, 1\}$$



Esquema nacimiento-sobrevivencia: $B3/S23$

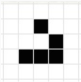
Juego de la Vida

$$G = \mathbb{Z}^2,\ A = \{0, 1\}$$



Glider

Glider



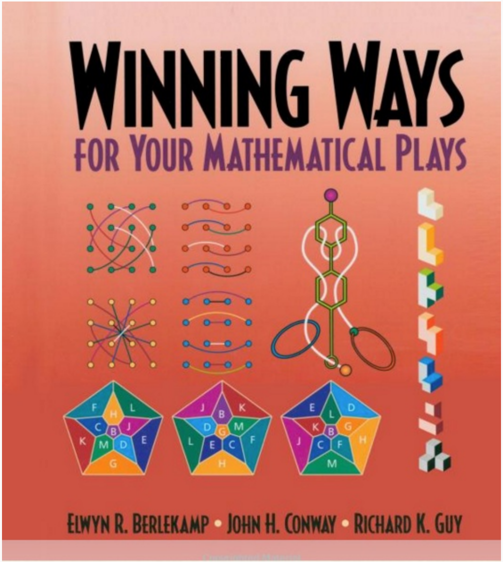
Definición Autómata Celular

Sea G un grupo y A un conjunto. *Un autómata celular sobre el grupo G y un alfabeto A es un mapeo* τ*:* $A^G \to A^G$ *que satisface la siguiente propiedad: existe un subconjunto finito* $S \subset G$ *y un mapeo* μ*:* $A^S \to A$ *tal que*

$$\tau(x)(g) = \mu((g^{-1}x)|s)$$

para toda $x \in A^G$ *y* $g \in G$*,l donde* $(g^{-1}\ x)|s$ *denota la restricción de la configuración* $g^{-1}x$ *a S.*

winning ways



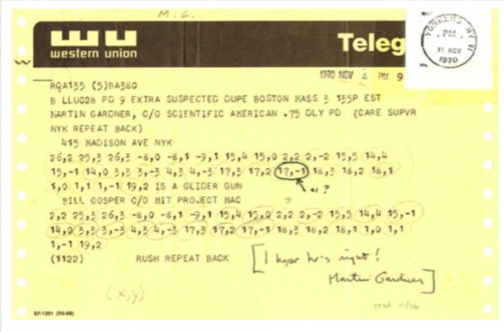
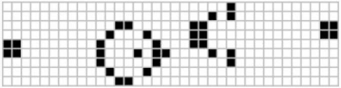

western union
Telegram
1970 NOV 4 PM 9
RQA135 (5)BA380
B LLU028 PD 9 EXTRA SUSPECTED DUPE BOSTON MASS 3 135P EST
MARTIN GARDNER, C/O SCIENTIFIC AMERICAN .75 DLY PD (CARE SUPVR
NYK REPEAT BACK)
415 MADISON AVE NYK
26,2 25,3 26,3 -8,0 -8,1 -9,1 15,4 15,0 2,2 2,-2 15,5 14,4
15,-1 14,0 3,3 3,-3 4,3 4,-3 17,3 17,2 17,-1 18,3 18,2 18,1
1,0 1,1 1,-1 19,2 IS A GLIDER GUN
BILL GOSPER C/O MIT PROJECT MAC
2,2 25,3 26,3 -8,0 -8,1 -9,1 15,4 15,0 2,2 2,-2 15,5 14,4 15,-1
14,0 3,3 3,-3 4,3 4,-3 17,3 17,2 17,-1 18,3 18,2 18,1 1,0 1,1
1,-1 19,2
(1122) RUSH REPEAT BACK
[I hope he's right!
Martin Gardner]
(x,y)

Fig. 2.1 Seminal telegram from Gosper to Gardner

Glider Gun

Glider Gun




SCIENTIFIC
AMERICAN
October 1970

Autómatas Celulares con el esquema Nacimiento-Sobrevivencia del Juego de la Vida

Esquema Nacimiento Sobrevivencia

- B35678/S5678
- B35/S236
- B27/S0
- B2/S7
- ...

R-Rule

Emannuel Sapin 2004

A New Universal Cellular Automaton Discovered by Evolutionary Algorithms

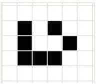
Emmanuel Sapin[1], Olivier Bailleux[1], Jean-Jacques Chabrier[1], and Pierre Collet[2]

[1] Université de Bourgogne, 9 av. A. Savary, B.P. 47870, 21078 Dijon Cedex, France
emmanuelsapin@hotmail.com {olivier.bailleux, jjchab}@u-bourgogne.fr
[2] Laboratoire d'Informatique du Littoral, ULCO, Calais, France
Pierre.Collet@Univ-Littoral.Fr

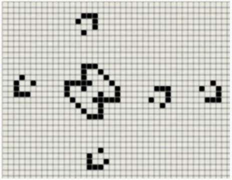
Abstract. In *Twenty Problems in the Theory of Cellular Automata*, Stephen Wolfram asks "how common computational universality and undecidability [are] in cellular automata." This papers provides elements of answer, as it describes how another universal cellular automaton than the Game of Life (*Life*) was sought *and found* using evolutionary algorithms. This paper includes a demonstration that consists in showing that the presented R automaton can both implement any logic circuit (logic universality) and a simulation of *Life* (universality in the Turing sense). All the elements of the evolutionary algorithms that were used to find R are provided for replicability, as well as the analytical description in R of a cell of *Life*.

Glider Sapin

Glider Sapin



Glider Gun Sapin



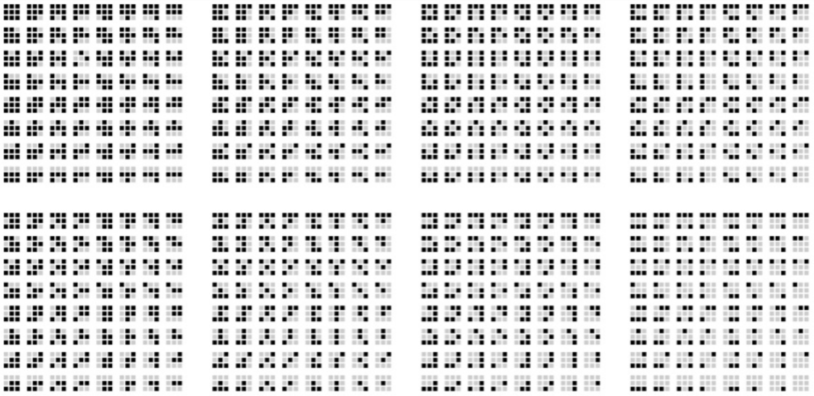
Búsqueda de AC con Computación Universal

Búsqueda de AC

Problema Combinatorio

Búsqueda de AC

μ



Número de reglas

$$\mu$$

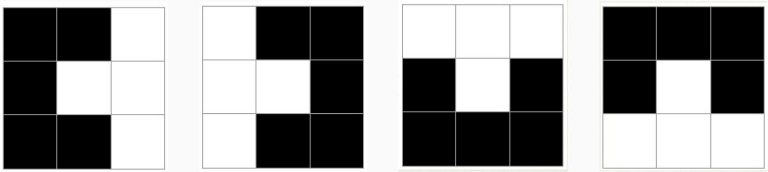
$$2^{512}$$

1340780792994259709957402499820584612
7479365820592393377723561443721764030
0735469768018742981669034276900318581
8581864860508537538828119465699464336
49006084096

Simetría

μ

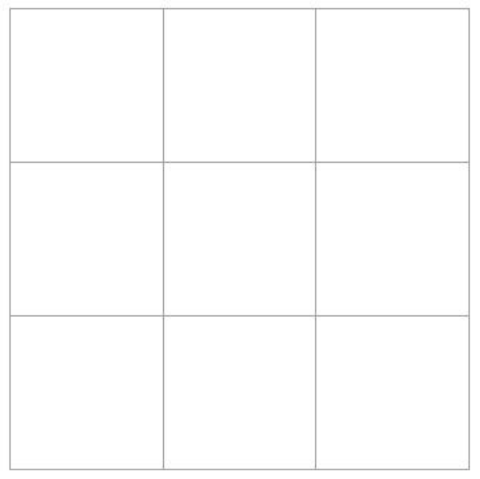
Equivalencias por Simetría



Equivalencias por Simetría

- 8 grupos con 1 elemento
- 8 grupos con 2 elementos
- 50 grupos con 4 elementos
- 36 grupos con 8 elementos
- 102 en total

$2^{102} = 5070602400912917605986812821504$



$2^{101} = 2535301200456458802993406410752$

clasificación

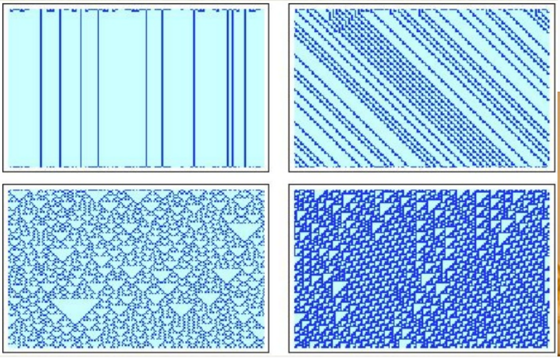
Physica 10D (1984) 1–35
North-Holland, Amsterdam

UNIVERSALITY AND COMPLEXITY IN CELLULAR AUTOMATA

Stephen WOLFRAM*
The Institute for Advanced Study, Princeton NJ 08540, USA

Cellular automata are discrete dynamical systems with simple construction but complex self-organizing behaviour. Evidence is presented that all one-dimensional cellular automata fall into four distinct universality classes. Characterizations of the structures generated in these classes are discussed. Three classes exhibit behaviour analogous to limit points, limit cycles and chaotic attractors. The fourth class is probably capable of universal computation, so that properties of its infinite time behaviour are undecidable.

clasificación



Physica D 42 (1990) 12–37
North-Holland

**COMPUTATION AT THE EDGE OF CHAOS:
PHASE TRANSITIONS AND EMERGENT COMPUTATION**

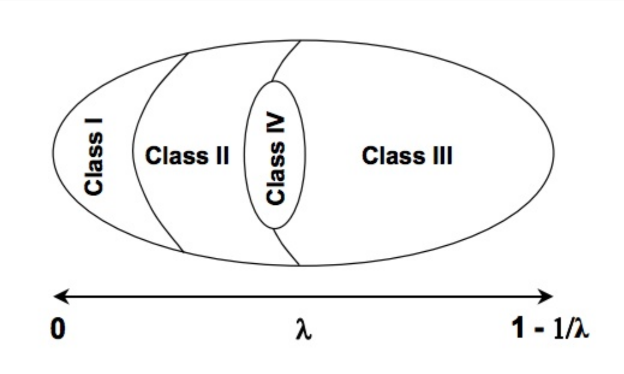
Chris G. LANGTON
Complex Systems Group, Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87454, USA

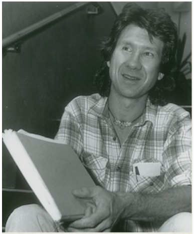
In order for computation to emerge spontaneously and become an important factor in the dynamics of a system, the material substrate must support the primitive functions required for computation: the transmission, storage, and modification of information. Under what conditions might we expect physical systems to support such computational primitives?

This paper presents research on cellular automata which suggests that the optimal conditions for the support of information transmission, storage, and modification, are achieved in the vicinity of a phase transition. We observe surprising similarities between the behaviors of computations and systems near phase transitions, finding analogs of computational complexity classes and the halting problem within the phenomenology of phase transitions.

We conclude that there is a fundamental connection between computation and phase transitions, especially second-order or "critical" transitions, and discuss some of the implications for our understanding of nature if such a connection is borne out.

parámetro lambda





Indecibilidad en la Clasificación de Autómatas Celulares

Complex Systems **2** (1988) 177–190

Undecidability of CA Classification Schemes*

Karel Culik II
Department of Computer Science, University of South Carolina, Columbia, SC 29208, USA

Sheng Yu
Department of Mathematical Sciences, Kent State University, Kent, OH 44242, USA

Abstract. Stephen Wolfram introduced the use of cellular automata as models of complex systems and proposed a classification of these automata based on their statistically observed behavior. We investigate various properties of these classes; in particular, we ask whether certain properties are effective, and we obtain several somewhat surprising results. For example, we show that it is undecidable whether all the finite configurations of a given cellular automaton eventually become quiescent. Consequently, it is undecidable to which class a given cellular automaton belongs, even when choosing only between the two simplest classes.

Shannon

Reprinted with corrections from *The Bell System Technical Journal,*
Vol. 27, pp. 379–423, 623–656, July, October, 1948.

A Mathematical Theory of Communication

By C. E. SHANNON

INTRODUCTION

THE recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist[1] and Hartley[2] on this subject. In the present paper we will extend the theory to include a number of new factors, in particular the effect of noise in the channel, and the savings possible due to the statistical structure of the original message and due to the nature of the final destination of the information.

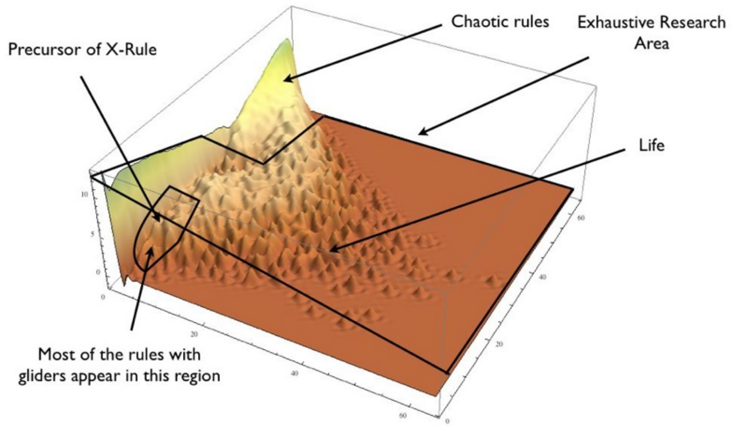
The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have *meaning*; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one *selected from a set* of possible messages. The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design.

If the number of messages in the set is finite then this number or any monotonic function of this number can be regarded as a measure of the information produced when one message is chosen from the set, all choices being equally likely. As was pointed out by Hartley the most natural choice is the logarithmic function. Although this definition must be generalized considerably when we consider the influence of the statistics of the message and when we have a continuous range of messages, we will in all cases use an essentially logarithmic measure.

Entropía

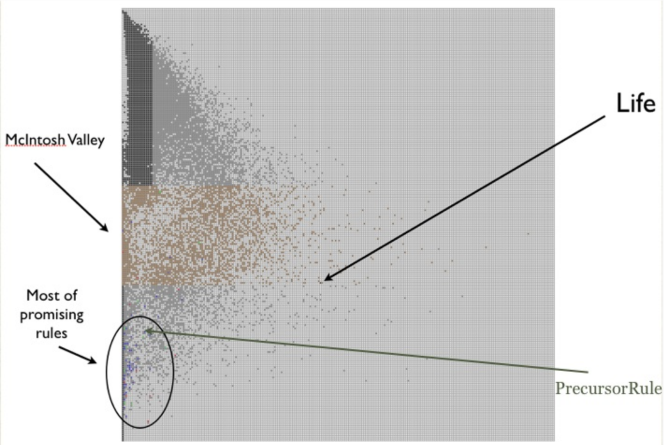
$$E = -\Sigma_{i=1}^{n} pLog(p)$$

Entropía



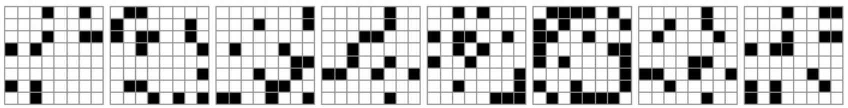
Precursor of X-Rule
Chaotic rules
Exhaustive Research Area
Life
Most of the rules with gliders appear in this region

Entropía

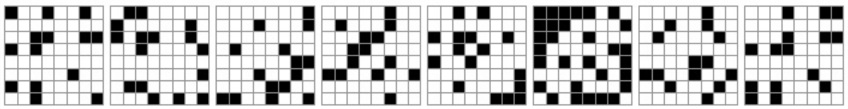

Life
McIntosh Valley
Most of
promising
rules
PrecursorRule

Precursor-Rule, X-Rule and Sayab-Rule

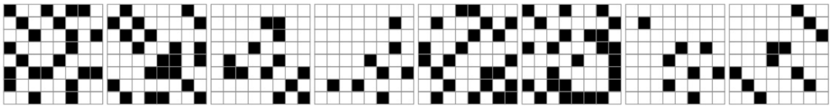
X-Rule



Precursor-Rule



Sayab-Rule



Computación Universal

Requisitos para tener Computación Universal en AC

1. Almacenamiento de datos.
2. Transmisión de datos.
3. Procesamiento de datos.

Procesamiento de datos

Post’s Functional Completeness Theorem

462

Notre Dame Journal of Formal Logic
Volume 31, Number 2, Spring 1990

Post’s Functional Completeness Theorem

FRANCIS JEFFRY PELLETIER and NORMAN M. MARTIN*

Abstract The paper provides a new proof, in a style accessible to modern logicians and teachers of elementary logic, of Post’s Functional Completeness Theorem. Post’s Theorem states the necessary and sufficient conditions for an arbitrary set of (2-valued) truth functional connectives to be expressively complete, that is, to be able to express every (2-valued) truth function or truth table. The theorem is stated in terms of five properties that an arbitrary connective may have, and claims that a set of connectives is expressively complete iff for each of the five properties there is a connective that lacks that property.

Establece que para procesar datos, solo es necesario tener las compuertas lógicas OR, NOT y AND. (Computación Lógica)

Requisitos para procesar datos en AC

1. “Glider-gun”
2. “Eater”
3. Convenientes coaliciones entre “Gliders”.

Requisitos para procesar datos en AC

Mostrar ejemplos en Golly

X-Rule

Journal of Cellular Automata, Vol. 0, pp. 1–34
Reprints available directly from the publisher
Photocopying permitted by license only

The X-Rule: Universal Computation in a Non-Isotropic Life-Like Cellular Automaton

JOSÉ MANUEL GÓMEZ SOTO[1,*] AND ANDREW WUENSCHE[†]

[1]*Universidad Autónoma de Zacatecas*
Unidad Académica de Matemáticas. Zacatecas, Zac. México
[2]*Discrete Dynamics Lab, London, UK*

Received: March 15, 2015. Accepted: March 26, 2015.

We present a new Life-like cellular automaton (CA) capable of logic universality – the X-rule. The CA is 2D, binary, with a Moore neighborhood and λ parameter similar to the game-of-Life, but is not based on birth/survival and is non-isotropic. We outline the search method. Several glider types and stable structures emerge spontaneously within X-rule dynamics. We construct glider-guns based on periodic oscillations between stable barriers, and interactions to create logical gates.

Keywords: Universality, cellular automata, glider-gun, logical gates.

Precursor-rule

X-Rule's Precursor is also Logically Universal

José Manuel Gómez Soto* *Universidad Autónoma de Zacatecas. Unidad Académica de Matemáticas. Zacatecas, Zac. México.*
Andrew Wuensche† *Discrete Dynamics Lab.*

Dedidated to the memory of Harold V. McIntosh our friend and teacher.
1929-2015

26 November 2016 (to appear in Journal of Cellular Automata)

Abstract

We re-examine the isotropic Precursor-Rule (of the anisotropic X-Rule[6]) and show that it is also logically universal. The Precursor-Rule was selected from a sample of biased cellular automata rules classified by input-entropy[11]. These biases followed most "Life-Like" constraints — in particular isotropy, but not simple birth/survival logic. The Precursor-Rule was chosen for its spontaneously emergent mobile and stable patterns, gliders and eaters/reflectors, but glider-guns, originally absent, have recently been discovered, as well as other complex structures from the Game-of-Life lexicon. We demonstrate these newly discovered structures, and build the logical gates required for universality in the logical sense.

keywords: universality, cellular automata, glider-gun, logical gates.

Sayab-rule

Logical Universality from a Minimal 2D Glider-Gun

José Manuel Gómez Soto* *Universidad Autónoma de Zacatecas. Unidad Académica de Matemáticas. Zacatecas, Zac. México.*
Andrew Wuensche† *Discrete Dynamics Lab.*

November 20, 2017

Abstract

To understand the underlying principles of self-organisation, and computation in cellular automata, it would be helpful to find the simplest form of the essential ingredients, glider-guns and eaters, because then the dynamics would be easier to interpret. Such minimal components emerge spontaneously in the newly discovered Sayab-rule, a binary 2D cellular automaton with a Moore neighborhood and isotropic dynamics. The Sayab-rule's glider-gun, which has just four live cells at its minimal phases, can implement complex dynamical interactions and the gates required for logical universality.

keywords: universality, cellular automata, glider-gun, logical gates.

www.conwaylife.com

ConwayLife.com

A community for Conway's Game of Life and related cellular automata

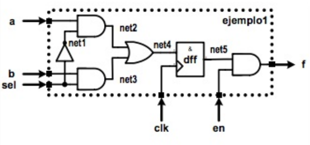
Gracias

Contacto:
José Manuel Gómez Soto
jmgomezuam@gmail.com
http://matematicas.reduaz.mx/∼jmgomez

Science


Mystery kidney disease strikes South Asia p. 24
Assessing young scientists' value p. 28
Pharmaceuticals made to order pp. 44 & 67
Science
$15
1 APRIL 2016
sciencemag.org
AAAS
Sequencing going ape
Higher resolution of the gorilla genome p. 52

Verilog




```
module ejemplo1(a,b,sel,clk,en,f);
//Inputs-Ouputs
input          a,b,sel,clk,en;
output         f;
wire           f;
//Descripción de los nodos internos
reg            net5;
wire           net1,net2,net3,net4;
// Descripción del diseño

endmodule
```

Cello

Cello

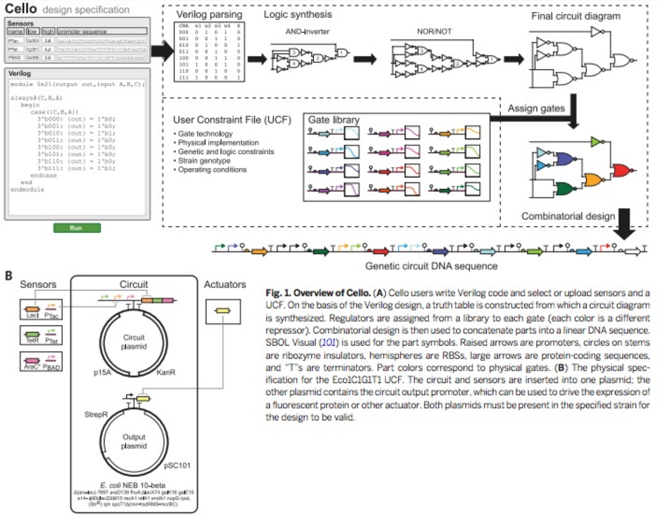


Fig. 1. Overview of Cello. (**A**) Cello users write Verilog code and select or upload sensors and a UCF. On the basis of the Verilog design, a truth table is constructed from which a circuit diagram is synthesized. Regulators are assigned from a library to each gate (each color is a different repressor). Combinatorial design is then used to concatenate parts into a linear DNA sequence. SBOL Visual (*101*) is used for the part symbols. Raised arrows are promoters, circles on stems are ribozyme insulators, hemispheres are RBSs, large arrows are protein-coding sequences, and "T"s are terminators. Part colors correspond to physical gates. (**B**) The physical specification for the Eco1C1G1T1 UCF. The circuit and sensors are inserted into one plasmid; the other plasmid contains the circuit output promoter, which can be used to drive the expression of a fluorescent protein or other actuator. Both plasmids must be present in the specified strain for the design to be valid.

Gracias

Contacto:
José Manuel Gómez Soto
jmgomezuam@gmail.com
http://matematicas.reduaz.mx/~jmgomez